COMMENTATIONES MATHEMATICAE UNIVERSITATIS CAROLINEA
25(1984), 171-179

CONVEX 4-VALENT POLYTOPES WITH PRESCRIBED TYPES OF FACES

MARIÁN TRENKLER

E.Jucovič [3] proved the following

Theorem. *If a sequence* $(p_k;\ k \geq 3)$ *of non-negative integers satisfies the conditions*

(i) $\sum_{k\geq 3}(4-k)p_k = 8,$ *and*

(ii) $p_4 \geq 13 + \sum_{k\geq 5}(3k-10)p_k$

then there exist a convex 4-valent 3-polytope **P** *which contains exactly* p_k k*-gons for all* k.

Such the sequence $(p_k;\ k \geq 3)$ is called 4-*realizable* and the polytope **P** is called its *realization*. This result was improved by *T.C.Enns* who showed that (ii) can be replaced by the condition

$$p_4 \geq 2\sum_{k\geq 5} p_k + \max\{k : p_k \neq 0\}$$

We prove the following theorem.

Theorem. *A sequence* $(p_k;\ k \geq 3)$ *of non-negative integers which satisfies the condition*

$$(i) \qquad \text{and} \qquad p_4 \geq \max\{k :\ p_k \equiv 1 \pmod 2\} - 2$$

is 4-realizable.

Our theorem is an improvement over previous theorems which required considerably langer values of p_4. If $\sum_{k\geq 5} p_k \leq 1$ our boundary is the best possible. In the paper are simple constructions of 4-valent polytopes with prescribed types of faces.

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1991 *Mathematics Subject Classification.* 52A25.
Key words and phrases. Convex polytope, k-gonal face, 4-valent vertex.